NUMERICAL SIMULATION OF FLOW DURING COMPRESSION OF CYLINDRICAL SAMPLES BY A GLANCING DETONATION WAVE

V. F. Lobanov

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The parameters of shock waves created in cylindrical samples of various materials during detonation of explosive charges surrounding them have been determined experimentally [1-3]. It was established that in a number of materials the reflection of a conical shock wave from the symmetry axis of a sample leads to the formation of a Mach triple shock-wave configuration which gives rise to a complex flow pattern in the region beyond the shock waves. Analytic study of irregular reflection is a complex problem. Solutions obtained under various assumptions about the nature of the flow are presented in papers reviewed in [4]. In the present paper, axisymmetric flow of detonation products (DP) and sample material in the region adjacent to the detonation front is determined from the solution of a two-dimensional, time-dependent problem in gasdynamics by the finite-difference method [5].

The system of conservation equations

$$\frac{\partial(\rho r)}{\partial t}+\frac{\partial(\rho u r)}{\partial x}+\frac{\partial(\rho v r)}{\partial r}=0;$$

$$\frac{\partial(\rho u r)}{\partial t}+\frac{\partial(p+\rho u^2)r}{\partial x}+\frac{\partial(\rho u v r)}{\partial r}=0;$$

$$\frac{\partial(\rho v r)}{\partial t}+\frac{\partial(\rho u v r)}{\partial x}+\frac{\partial(p+\rho v^2)r}{\partial r}=p;$$

$$\frac{\partial\left[\rho r\left(e+\frac{u^2+v^2}{2}\right)\right]}{\partial t}+\frac{\partial\left[\rho u\left(e+\frac{p}{\rho}+\frac{u^2+v^2}{2}\right)\right]r}{\partial x}+\frac{\partial\left[\rho v\left(e+\frac{p}{\rho}+\frac{u^2+v^2}{2}\right)r\right]}{\partial r}=0,$$

where u and v are the axial and radial components of the velocity vector, p, ρ, and e are the pressure, density, and specific internal energy, and x and r are the coordinates of the points, was supplemented by the equation of state for the detonation products

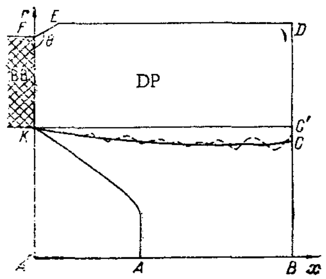

Fig. 1

$$p=A\rho^{\gamma},\ A,\ \gamma-\text{const},$$

and by the equation of state for the sample material in the form

$$p=\beta\rho e+c_0^2(\rho-\rho_0),\ \beta,\ \rho_0,\ c_0-\text{const}.$$

The calculation was performed in the region ABCDEFK (Fig. 1), where AB is the symmetry axis; FK is the detonation front, which is assumed plane, perpendicular to the generators of the charge, and propagating

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at a constant velocity D. The angle θ is found from the condition for the determination of the limiting characteristic of Prandtl–Meyer flow,

$$\cos\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\theta\right)=0,$$

which is realized in the neighborhood of the point F for dispersion of DP into a vacuum; $|FE| = hx/\cos(\theta-\pi/2)$, where hx, a step along x in the finite-difference mesh, is constant in the DP region; FE, ED, and BD are the outer boundaries of the region through which efflux of DP and material from the region of calculation occurs. During the calculation, the position of the boundaries KC and KA shifted from the arbitrarily assigned initial positions KC' and KA' to the corresponding contact discontinuity between DP, sample material, and the leading shock wave.

The region of computation was subdivided into N layers along the x axis and M + K layers along r (M and K are the number of layers in the sample and in DP). The coordinates of the mesh points of the moving finite-difference mesh were determined from the expressions (numbering from the origin at A')

$$\begin{cases} x_{i,j}=x_{0,j}+i\,(x_{N,j}-x_{0,j})/N, \\ r_{i,j}=jr_{i,M}/M, \\ 0\leqslant i\leqslant N,\quad 0\leqslant j\leqslant M; \end{cases}$$

$$\begin{cases} x_{i,j}=x_{N,j}(i/N), \\ r_{i,j}=r_{i,M}+(j-M)(r_{i,(M+K)}-r_{i,M})/K, \\ 0\leqslant i\leqslant N;\ M+1\leqslant j\leqslant(M+K); \end{cases}$$

$$x_{N,j}=x_1;\ 0\leqslant j\leqslant(M+K);\ r_{0,j}=j(r_2/M);\ 0\leqslant j\leqslant M;$$

$$r_{i,(M+K)}=r_0;\ 1\leqslant i\leqslant N;\ x_{i,M}=i(x_1/N);\ 0\leqslant i\leqslant N;\ r_{0,(M+K)}=r_1,$$

where x_1, r_0, r_1, and r_2 are constants, $r_0 = r_1 + x_1 \tan[\theta-(\pi/2)]/2$, and $r_{i,M}$ and $x_{0,j}$ ($0 \leq i \leq N$, $0 \leq j \leq M$) were determined at mesh points on KC and KA by displacement along r and x at velocities calculated by interpolation of the velocities of adjacent segments with weights proportional to the lengths of those segments in accordance with expressions similar to those given in [5].

The values of the gasdynamic quantities on FK were assumed equal to values of the corresponding quantities in the Chapman–Jouguet state. The pressure on FE and the radial component of the velocity on AB were assumed to be zero. The values of the parameters on the boundaries BD and ED were assumed equal to the values of the corresponding parameters in the internal cells of the region. The boundary conditions on KA and KC were calculated from iteration formulas for the calculation of detonation decay [5] altered for an equation of state in the form (1). Values of the parameters on the boundaries of internal cells were determined from approximate formulas for the calculation of weak ("acoustic") decay of a detonation.

The initial values of gasdynamic quantities were assumed independent of radius and were found from the expressions

$$\begin{aligned} u(x)&=\alpha[c_{C\text{-}J}+\delta_1(U-c_{C\text{-}J})x/x_1]+\delta D; \\ \rho(x)&=\alpha[\rho_{C\text{-}J}+\delta_2(\rho_{C\text{-}J}-\rho_1)x/x_1]+\delta\rho_0; \\ e(x)&=\alpha[e_{C\text{-}J}+\delta_3(e_{C\text{-}J}-e_1)x/x_1]; \\ v(x)&=0, \end{aligned}$$

where $\alpha = 1$ and $\delta = 0$ in the region occupied by DP; $\alpha = 0$ and $\delta = 1$ in the region ABCK; c_{C-J}, ρ_{C-J}, and e_{C-J} are the velocity of sound, density, and internal energy in the Chapman–Jouguet state; δ_1, δ_2, δ_3, U, ρ_1, and e_1 are constants chosen for approximation of the solution to the one-dimensional problem on the propagation of a detonation in a tube with constant cross section and rigid walls.

The basic calculations were performed for explosive parameter values D = 7.65 km/sec, P_{C-J} = 250 kbar, ρ_{C-J} = 2.25 g/cm^3, γ = 2.75, c_{C-J} = 5.61 km/sec, and D = 6.60 km/sec, P_{C-J} = 128 kbar, ρ_{C-J} = 1.5 g/cm^3, γ = 2.75, c_{C-J} = 4.84 km/sec. In all the versions of the calculations described, $r_1/r_2 = 4$ (ratio of external charge radius to internal charge radius). For the selected ratio of radii, replacement of the boundary condition on FD ($\theta = \pi/2$) by the condition v/FD = 0 leads to an insignificant change in the gasdynamic parameters of material flow.

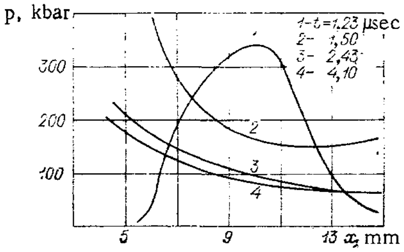

Fig. 2

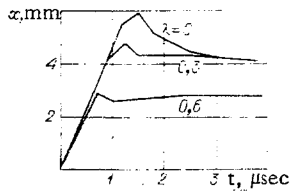

Fig. 3

The parameters β, ρ_0, and c_0 in Eq. (1) were chosen such that the selected equation of state was a model of an actual medium under the conditions of the problem considered. For example, for water it was assumed $\rho_0 = 1\ g/cm^3$, $c_0 = 2.15$ km/sec, and $\beta = 2$; for aluminum, $\rho_0 = 2.79\ g/cm^3$, $c_0 = 5.25$ km/sec, and $\beta = 2.75$; for magnesium, $\rho_0 = 1.725\ g/cm^3$, $c_0 = 4.45$ km/sec, and $\beta = 2.75$.

The dependence of the pressure distribution on the sample axis at various times (Fig. 2) and also the (x, t) diagram for individual mesh points on the boundary KA (Fig. 3) (values of r at these mesh points remained unchanged in accordance with the algorithm for the construction of the mesh, $\lambda = r/r_2$) characterize the process for the establishment of KA in a position corresponding to the position of the leading shock wave and producing a steady-state flow mode.

The determination of the steady-state position of the contact discontinuity is complicated by its instability. The interface between DP and sample material, the coordinates of which were obtained by averaging over the time interval $\tau = 0.6r_2/D$, is denoted by the solid line KC in Fig. 1; the dashed line denotes the interface at a given point in time. The instability of such a tangential discontinuity in ideal gases has been demonstrated [6] and the existence of instability of the DP–metal interface is shown by waves which remain on the surface of metal samples in experiments on loading by a detonation wave glancing along the surface. The wavelength of the perturbation depends linearly on the mesh step, covers 4-5 computing intervals, and is apparently determined by finite-difference "blurring" of the reflected shock wave, the emergence of which at the interface creates the instability. The question of the relation between the observed and actual stabilities requires additional study.

The pattern of the distribution of gasdynamic parameters in water (D = 6.60 km/sec) is shown in Fig. 4 at the time of establishment of a steady-state flow mode. Lines of equal density are shown by the solid lines and isobars are indicated by the dashed lines. The isobars in the neighborhood of the leading shock wave are not plotted in Fig. 4, since their position coincides with the lines of equal density. The pressure in the region beyond the linear portion of the shock wave adjacent to the axis is ≈210 kbar and is ≈50 kbar near the corner. The lengths of the vectors are proportional to velocity. (The velocity near the point A is ≈3 km/sec.) The numbers give the numerical values (ρ in g/cm^3, p in kbar) and the remaining notation is the same as in Fig. 1.

A qualitative analysis of the flow pattern makes it possible to conclude that there is within the region a weak shock wave and a jet of material in the paraxial zone with a specific energy considerably greater than the specific energy in the peripheral layers of the material. Determination of the exact position of the reflected wave and of the tangential discontinuity between material passing through the straight shock wave and the two slanted waves is difficult because of their "blurring" by finite-difference effects over several computing intervals.

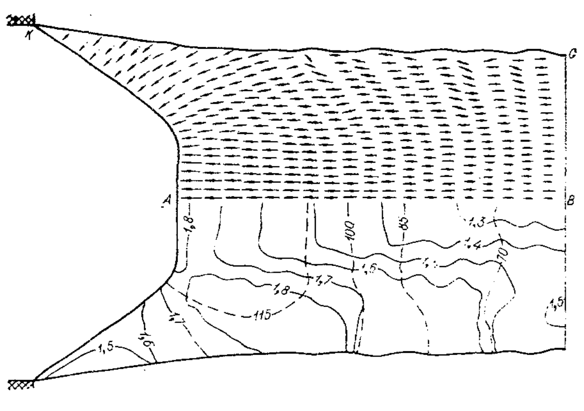

Fig. 4

The shape of the leading shock wave in water for D = 7.65 km/sec is shown in Fig. 5. (curve 3). The difference between shock waves when D = 6.60 km/sec (curve 4) and 7.65 km/sec in the neighborhood of the corner K is so insignificant that they duplicate one another in the graph. The difference in the relative fraction of the linear portion of the shock wave adjacent to the axis, the so-called Mach disk, is more marked.

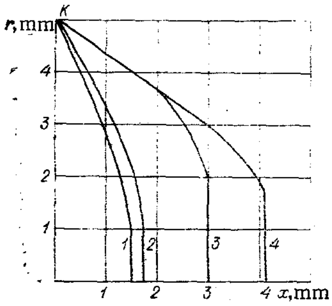

Fig. 5

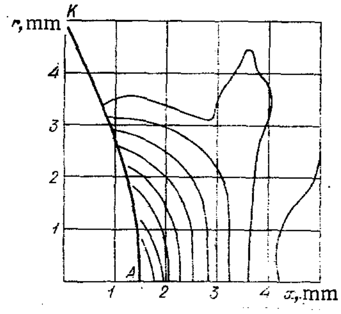

Fig. 6

For aluminum (curve 1) and magnesium (curve 2) samples at D = 6.60 km/sec, the shape of the leading shock wave is nearly parabolic (Fig. 5) and is qualitatively similar to that recorded for aluminum [3]. Analysis of the values of the gasdynamic quantities shows that a flow mode with a velocity insignificantly greater than the velocity of sound in the material is achieved beyond the front of the leading shock wave only in a small neighborhood of the point K. Therefore, the emergence of the reflected shock wave, the creation of which is only possible in this region on the boundary between DP and sample material, and the resultant instability of the contact discontinuity begin directly beyond the detonation front. Determination of the position of the reflected shock wave is practically impossible because of the "blurring" and marked oscillation in the values of the gasdynamic quantities in the cells adjacent to the contact discontinuity. The shape of the isobars beyond the front KA of the leading shock wave in aluminum is shown in Fig. 6. Numerical values of the pressure are 100 kbar for the rightmost isobar and 220 kbar for the leftmost isobar on the axis. The isobars in magnesium are of a qualitatively similar form.

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